Problem 8.2

a) $$E^i = \hat{y}\, 8\cos(6\pi\times10^9 t - 30\pi x) \ (\text{V/m})$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{\eta_0}{\sqrt{2.25}} = \frac{377}{1.5} = 251.33\ \Omega$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{377}{2} = 188.5\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{2} - \frac{1}{1.5}}{\frac{1}{2} - \frac{1}{1.5}} = -0.143$$

$$\tau = 1 + \Gamma = 1 - 0.143 = 0.857$$

$$E^r = \Gamma E^i = -1.14\,\hat{y}\cos(6\pi\times10^9 t + 30\pi x) \ (\text{V/m})$$

$$E_1 = E^i + E^r = \hat{y}\left[8\cos(6\pi\times10^9 t - 30\pi x) - 1.14\cos(6\pi\times10^9 t + 30\pi x)\right] \frac{\text{A}}{\text{m}}$$

$$H^i = \hat{z}\frac{8}{\eta_1}\cos(6\pi\times10^9 t - 30\pi x) = \hat{z}\, 31.83\cos(6\pi\times10^9 t - 30\pi x) \frac{\text{mA}}{\text{m}}$$

$$H^r = \hat{z}\frac{1.14}{\eta_1}\cos(6\pi\times10^9 t + 30\pi x) = \hat{z}\, 4.54\cos(6\pi\times10^9 t + 30\pi x) \left(\frac{\text{mA}}{\text{m}}\right)$$

$$H_1 = H^i + H^r$$
$$= \hat{z}\left[31.83\cos(6\pi\times10^9 t - 30\pi x) + 4.54\cos(6\pi\times10^9 t + 30\pi x)\right] \frac{\text{mA}}{\text{m}}$$

$$k_1 = \omega\sqrt{\mu\varepsilon_1} \quad \text{and} \quad k_2 = \omega\sqrt{\mu\varepsilon_2} \Rightarrow k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\,k_1 = \sqrt{\frac{4}{2.25}}\,30\pi = 40\pi \ \left(\frac{\text{rad}}{\text{m}}\right)$$

$$E_2 = E^t = \hat{y}\, 8\tau\cos(6\pi\times10^9 t - 40\pi x) = \hat{y}\, 6.86\cos(6\pi\times10^9 t - 40\pi x) \ \left(\frac{\text{V}}{\text{m}}\right)$$

$$H_2 = H^t = \hat{z}\frac{8\tau}{\eta_2}\cos(6\pi\times10^9 t - 40\pi x) = \hat{z}\, 36.38\cos(6\pi\times10^9 t - 40\pi x) \left(\frac{\text{mA}}{\text{m}}\right)$$

b)

$$S^{i}_{av} = \hat{x}\,\frac{8^2}{2\eta_1} = \frac{64}{2\times 251.33} = \hat{x}\ 127.3 \quad (mW/m^2)$$

$$S^{r}_{av} = -|\Gamma|^2 S^{i}_{av} = -\hat{x}\,(0.143)^2 \times 0.127 = -\hat{x}\ 2.6 \quad \left(\frac{mW}{m^2}\right)$$

$$S^{t}_{av} = \frac{|E^{t}_{0}|^2}{2\eta_2}$$

$$= \hat{x}\,\tau^2\,\frac{(8)^2}{2\eta_2} = \hat{x}\,\frac{(0.86)^2\,64}{2\times 188.5} = \hat{x}\ 124.7 \quad \left(\frac{mW}{m^2}\right)$$

Problem 8.4

a) $k_1 = \frac{\omega}{c} = \frac{2\pi \times 2\times10^8}{3\times10^8} = \frac{4\pi}{3}$ rad/m

$k_2 = \frac{\omega}{u_{p_2}} = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{4\pi}{3}\sqrt{4} = \frac{8\pi}{3}$ $\frac{\text{rad}}{\text{m}}$

$\tilde{E}^i = a_0 (\hat{x} + \hat{y} e^{j\pi/2}) e^{-jkz} = a_0 (\hat{x} + j\hat{y}) e^{-jkz}$

$E^i(z,t) = \hat{x} a_0 \cos(\omega t - kz) - \hat{y} a_0 \sin(\omega t - kz)$,

$|E^i| = [a_0^2 \cos^2(\omega t - kz) + a_0^2 \sin^2(\omega t - kz)]^{\frac{1}{2}} = 5$ (V/m)

$\rightarrow \quad \tilde{E}^i = 5(\hat{x} + j\hat{y}) e^{-j4\pi z/3}$ (V/m)

b) $\eta_1 = \eta_0 = 120\pi$ (Ω), $\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2} = 60\pi$ (Ω)

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60}{180} = -\frac{1}{3}$ $\quad \tau = 1 + \Gamma = \frac{2}{3}$

c)
$\tilde{E}^r = 5\Gamma(\hat{x} + j\hat{y}) e^{jk_1 z} = -\frac{5}{3}(\hat{x} + j\hat{y}) e^{j4\pi z/3}$ (V/m)

$\tilde{E}^t = 5\tau(\hat{x} + j\hat{y}) e^{-jk_2 z} = \frac{10}{3}(\hat{x} + j\hat{y}) e^{-j8\pi\frac{z}{3}}$ (V/m)

$\tilde{E}_1 = \tilde{E}^i + \tilde{E}^r = 5(\hat{x} + j\hat{y})[e^{-j4\pi\frac{z}{3}} - \frac{1}{3} e^{j4\pi z/3}]$ (V/m)

d) % reflected power $= 100 \times |\Gamma|^2 = \frac{100}{9} = 11.11\%$

% transmitted power $= 100 \times |\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times (\frac{2}{3})^2 \times \frac{120\pi}{60\pi} = 88.89\%$

Problem 8.6

a) $\eta_1 = \eta_0 = 120\pi \ (\Omega)$ $\quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_{r_2}}} = 20\pi \ (\Omega)$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71$$

$\rightarrow |\Gamma| = 0.71 \ , \ \theta_\eta = 180^\circ$

b)
$$S_{av}^i = \frac{|E_0^i|^2}{2\eta_1} = \frac{(50)^2}{2 \times 120\pi} = 3.32 \quad (W/m^2)$$

$$S_{av}^r = |\Gamma|^2 S_{av}^i = (0.71)^2 \times 3.32 = 1.67 \quad (W/m^2)$$

c)

medium 1 (air):

$$\lambda_1 = \frac{c}{f} = \frac{3\times10^8}{5\times10^7} = 6 \text{ m}$$

$$l_{max} = \frac{\theta_r \lambda_1}{4\pi} = \frac{\pi \times 6}{4\pi} = 1.5 \text{ m}$$

$$l_{min} = l_{max} - \frac{\lambda_1}{4} = 1.5 - 1.5 = 0 \quad \text{m} \quad \text{(at the boundary)}$$

Problem 8.12

$$\lambda = \frac{\lambda_o}{\sqrt{\varepsilon_r}} = \frac{0.61}{\sqrt{1.44}} = 0.508\ \mu m$$

The light would appear green

Problem 8.18

For violet: $n_v = 1.71 - \frac{4}{30} \times 0.4 = 1.66$

$$\sin\theta_2 = \frac{\sin\theta}{n_v} = \frac{\sin 50}{1.66} \rightarrow \theta_2 = 27.48^\circ$$

from the geometry of triangle ABC

$$180^\circ = 60^\circ + (90 - \theta_2) + (90^\circ - \theta_3)$$

$$\rightarrow \theta_3 = 60 - \theta_2 = 60 - 27.48^\circ = 32.52^\circ$$

$$\sin\theta_4 = n_v \sin\theta_3 = 1.66 \sin 32.52^\circ = 0.89$$

$$\theta_4 = 63.18^\circ$$

For red, $n_r = 1.71 - \frac{4}{30} \times 0.7 = 1.62$

$$\theta_2 = \sin^{-1}\left[\frac{\sin 50^\circ}{1.62}\right] = 28.22^\circ$$

$$\theta_3 = 60^\circ - 28.22^\circ = 31.78^\circ$$

$$\theta_4 = \sin^{-1}[1.62 \sin 31.78^\circ] = 58.56^\circ$$

So angular dispersion $= 63.18^\circ - 58.56^\circ = 4.62^\circ$

Problem 8.26

a) $\tilde{E}^i = \hat{y}\, 20 e^{-j(3x+4z)}$ (V/m)

Since E^i is along $\hat{y}$, and it is perpendicular to the plane of incidence, the wave is perpendicularly polarized

b) Eq (8.48a) $\rightarrow$ argument of the exponential

$$\rightarrow -jk_1(x\sin\theta_i + z\cos\theta_i) = -j(3x+4z)$$

$$\rightarrow k_1 \sin\theta_i = 3 \quad , k_1\cos\theta_i = 4$$

$$\rightarrow tg\theta_i = \frac{3}{4} \rightarrow \theta_i = 36.87^\circ$$

$$k_1 = \sqrt{3^2+4^2} = 5 \text{ (rad/m)}$$

$$\omega = u_p k = ck = 3\times10^8 \times 5 = 1.5\times10^9 \text{ (rad/s)}$$

c) $\eta_1 = \eta_0 = 377\ \Omega$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{\eta_0}{2} = 188.5\ \Omega$$

$$\theta_t = \sin^{-1}\left[\frac{\sin\theta_i}{\sqrt{\varepsilon_{r_2}}}\right] = \sin^{-1}\left[\frac{\sin 36.87^\circ}{\sqrt{4}}\right] = 17.46^\circ$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.41$$

$$\tau_\perp = 1 + \Gamma_\perp = 0.59$$

Eq (8.49a) and $E_0^r = \Gamma_\perp E_0^i$

$$\tilde{E}^r = -\hat{y}\, 8.2 e^{-j(3x-4z)}$$

$$\tilde{H}^r = (\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\frac{8.2}{\eta_0} e^{-j(3x-4z)}$$

$\theta_i = \theta_r$ and z-direction has been reversed

$$\rightarrow E^r = \mathrm{Re}[\tilde{E}^r e^{j\omega t}] = -\hat{y}\, 8.2\cos(1.5\times10^9 t - 3x + 4z) \quad (\mathrm{V/m})$$

$$H^r = (\hat{x}\, 17.4 + \hat{z}\, 13.06)\cos(1.5\times10^9 t - 3x + 4z)$$

d) medium 2,

$$k_2 = k_1\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5\sqrt{4} = 20 \quad (\mathrm{rad/m})$$

$$\theta_t = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i\right] = \sin^{-1}\left[\frac{1}{2}\sin 36.87^\circ\right] = 17.46^\circ$$

exponent of E^t and H^t is

$$-jk_2(x\sin\theta_t + z\cos\theta_t) = -j10(x\sin 17.46^\circ + z\cos 17.46)$$

$$= -j(3x + 9.54z)$$

$$\Rightarrow \tilde{E}^t = \hat{y}\, 20\times0.59\, e^{-j(3x+9.54z)}$$

$$\tilde{H}^t = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)\frac{20\times0.59}{\eta_2} e^{-j(3x+9.54z)}$$

$$E^t = \mathrm{Re}[\tilde{E}^t e^{j\omega t}] = \hat{y}\, 11.8\cos(1.5\times10^9 t - 3x - 9.54z) \quad (\mathrm{V/m})$$

$$H^t = (-\hat{x}\cos 17.46^\circ + \hat{z}\sin 17.46^\circ)\frac{11.8}{188.5}\cos(1.5\times10^9 t - 3x - 9.543z)$$

$$= (-\hat{x}\,59.72 + \hat{z}\,18.78)\cos(1.5\times10^9 t - 3x - 9.543z)\ \left(\frac{mA}{m}\right)$$

e) $$S_{av}^t = \frac{|E_0^t|^2}{2\eta_2} = \frac{(11.8)^2}{2\times 188.5} = 0.36\ (W/m^2)$$

Problem 8.33

a) $$\Gamma_{\parallel} = \frac{-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_i}}$$

$$= \frac{-2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}}{2.6\cos 50^\circ + \sqrt{2.6 - \sin^2 50^\circ}} = -0.08$$

$$R_{\parallel} = |\Gamma_{\parallel}|^2 = (0.08)^2 = 6.4\times10^{-3}$$

$$T_{\parallel} = 1 - R_{\parallel} = 0.9936.$$

b) $$P_{\parallel}^i = \frac{|E_{\parallel 0}^i|^2}{2\eta_1} A\cos\theta_i = \frac{(10)^2}{2\times 120\pi}\times\cos 50^\circ = 85\ mW$$

$$P_{\parallel}^r = R_{\parallel} P_{\parallel}^i = (6.4\times10^{-3})\times 0.085 = 0.55\ mW$$

$$P_{\parallel}^t = T_{\parallel} P_{\parallel}^i = 0.9936\times 0.085 = 84.45\ mW$$